Current Account Models

1. A Model without Uncertainty

Suppose a representative agent for a small open economy needs to make the following optimal decision to maximize her life-time utility:

$$Max.U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

Where, β is the discount factor and C_s is the consumption at period s. The representative agent is subject to the following constraint:

$$CA_t = B_{t+1} - B_t = rB_t + Y_t - C_t - G_t - I_t$$

Where, CA_t is the current account at time t, B_t is the net foreign asset. This budget constraint basically states that any change in net foreign asset results from interest earnings on foreign bond (rB_t) and any current surplus ($Y_t - C_t - G_t - I_t$).

Here I_t is the current investment and the capital stock evolves as below:

$$K_{t+1} = I_t + K_t$$

Where, the depreciation rate is ignored. The production function is given by:

$$Y_t = A_t F(K_t)$$

Here we leave all other factors including labor factors and technology improvements to A_t and the function F satisfies the Inada conditions: $F(0) = 0$, F is continuously differentiable, $F' > 0$, $F'' < 0$, $\lim_{K \to 0} F'(K) = \infty$ and $\lim_{K \to \infty} F'(K) = 0$.

2. Optimization Solution:

Set up the Lagrange and obtain the first order conditions:

$$u'(C_s) = \beta(1+r)u'(C_{s+1}) \text{ (Euler equation)}$$

And the capital allocation is decided by:

$$A_{s+1}F'(K_{s+1}) = r$$

Transversility condition:

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0$$

3. A Useful Example:

Consider the case $\beta = 1/(1+r)$, we may have $C_s = C_{s+1}$ and plug this into the budget constraint to derive:

$$C_t = rB_t + \frac{r}{1+r}\cdot\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s - I_s - G_s\right)$$

Define *constant* $\tilde{X}_t$ such that:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tilde{X}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

Therefore,

$$\tilde{X}_t = \frac{r}{1+r}\cdot\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

Substitute this and the consumption solution into budget constraint to get:

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t)$$

Remarks:

1. Current account is determined by the deviation from current net output from its permanent level;
2. The current account increases if the current income increases more than its permanent level;
3. The current account decreases if the current investment or government spending increases more than its permanent level.

4. A Stochastic Current Account Model:

Facing uncertainty, the agent maximizes her life-time expected utility:

$$Max.E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}u(C_s)\right\},$$

subject to the same budget constraint as above.

Euler equation:

$$E_t\{u'(C_s)\} = \beta(1+r)E_t\{u'(C_{s+1})\}$$

Now consider $\beta = 1/(1+r)$ along with the Linear-Quadratic utility model:

$$u(C) = C - \frac{a_0}{2}C^2, \qquad a_0 > 0$$

With this particular utility model it is easy to show that:

$$E_t(C_{t+1}) = C_t$$

This implies the consumption is a random walk, as originally suggested by Hall (1987):

$$C_{t+1} = C_t + \varepsilon_{t+1}, \qquad E_t[\varepsilon_{t+1}] = 0$$

Plug this result into the budget constraint to derive:

$$C_t = rB_t + \frac{r}{1+r} \cdot \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \cdot E_t \left(Y_s - I_s - G_s \right)$$

We end up with a similar result as above but for uncertainty world. The same remarks follow. It is, however, an empirical issue to determine how much of a particular rise (fall) in $Y_t - G_t - I_t$ reflects a permanent change and how much a transitory change. This relates to a very interesting and ever-lasting debate in applied macroeconomics literature.

5. Case I: Output Y_t Is Stationary.

Consider the following simple *AR(1)* process for the output:

$$Y_{t+1} - \overline{Y} = \rho \cdot (Y_t - \overline{Y}) + e_{t+1}, \qquad 0 \le \rho < 1$$

Note this implies:

$$Y_{t+1} = \overline{Y} + \sum_{-\infty}^{t+1} \rho^{t+1-s} e_s$$

Suppose $G = I = 0$, this implies:

$$C_t = rB_t + \overline{Y} + \frac{r(Y_t - \overline{Y})}{1+r-\rho}$$

Notice that this gives a Keynesian consumption function:

The Marginal Propensity to Consume (MPC) is $0 < \frac{r}{1+r-\rho} < 1$.

Remarks:

1. When the output is stationary, one unit increase in current output increases consumption by $\frac{r}{1+r-\rho}$. As a result the current account increases by $\frac{1-\rho}{1+r-\rho}$;

2. Note if $\rho = 1$, i.e., the output is a random walk, MPC = 1. Any increase in current output leads to the same increase in consumption and no change in current account.

6. Case II: Output Y_t Is Difference Stationary.

Consider the following simple *AR(1)* process for the output growth:

$$Y_{t+1} - Y_t = \rho \cdot (Y_t - Y_{t-1}) + e_{t+1}, \qquad 0 \le \rho < 1$$

Now this implies:

$$Y_{t+1} = Y_t + \sum_{-\infty}^{t+1} \rho^{t+1-s} e_s$$

Plug this back into the budget constraint to get:

$$C_{t+1} - C_t = \frac{1+r}{1+r-\rho} \cdot e_{t+1}$$

Therefore it is straightforward to show:

$$CA_{t+1} - CA_t = -\frac{\rho}{1+r-\rho} \cdot e_{t+1}$$

Remarks:

1. Consumption increases more than current output since $\frac{1+r}{1+r-\rho} \ge 1$.

2. Current account decreases when current output increases.